Solid-Phase Creep During the Expression of Palm-Oil Filter Cakes

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For an adequate model of the processes of compressible cake filtration and mechanical expression, permeability and compressibility data are required. Experimental and modeling results of the creep behavior of palm-oil filter cakes at constant and time-dependent pressures are presented. Creep curves of palm-oil filter cakes at constant pressures cannot be modeled with linear viscoelastic models. Modeling with a modified form of the empirical equation of Nutting gives satisfactory results. This modification does not lead to unrealistic values of the porosity at extreme conditions, contrary to the original form of the equation of Nutting. Creep curves at time-dependent pressures were modeled with two nonlinear viscoelastic models, which describe the time-dependent creep behavior as a function of the pressure history and creep curves at constant pressures. Modeling with the strain-hardening model provides the best porosity predictions.

Introduction

The viscoelastic behavior of some materials, like sewage sludge (La Heij, 1994) and clays (Shirato et al., 1978) plays an important role during solid–liquid separation processes. The consolidation of soils is another example where viscoelastic behavior is important (Den Haan, 1994).

For edible fats in food products, a very specific melting range is often desired. In most cases, the unmodified edible oils cannot be used directly in food products. A modification of the melting range is therefore necessary. An attractive method is partial crystallization from the melt, followed by cake filtration and, in most cases, subsequent expression (Deffense, 1985). In the edible-oil and -fat industry this technique is known as the dry-fractionation process. The major disadvantage of this process, however, is the low separation efficiency of the filtration and expression steps. A predictive model of the filtration step and particularly the expression step is therefore an important tool for the optimization of the dry-fractionation process.

During filtration and mechanical expression the external load is carried partly by the liquid and the remainder by the solids. Therefore, besides a liquid pressure gradient, a solid compressive pressure gradient is also induced. Because the local porosity depends on the solid compressive pressure, the pressure gradient will automatically give rise to a porosity gradient. From this, it follows that during consolidation there are time-dependent gradients of liquid pressure, solid compressive pressure, and porosity. In the case of equilibrium consolidation these gradients are nonexistent and the complete external load is carried by the solids.

Compressible cake filtration and mechanical expression are usually modeled with a theory based on mass and momentum balances. A general approach for the modeling of compressible cake filtration and expression was given by Tiller and Yeh (1987) and Stamatakis and Tien (1991). This approach leads to an equation with a material independent part, which is based on the mass and momentum balances of the liquid and solid phase, and with a material dependent part, which is based on the permeability and the compressibility of the bulk material. For the modeling of compressible-cake filtration and expression, information about the bulk permeability and compressibility must therefore be available. The bulk compressibility is usually defined as follows (Moore, 1972):

$$\beta = -\frac{1}{V}\left(\frac{\partial V}{\partial p_s}\right)_T = -\left(\frac{\partial \epsilon}{\partial p_s}\right)_T, \tag{1}$$

where

β = bulk compressibility, Pa^{-1}
V = cake volume, m^3
T = temperature, K
ϵ = natural volume strain
p_s = solid compressive pressure, defined as the ratio of the force on the solids in a cross section of the cake and the filter area, Pa

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The natural volume strain ϵ is defined by the relative volume change of the solid phase (Barends, 1980):

$$d\epsilon = \frac{dV}{V} \Rightarrow \epsilon = \ln\left(\frac{V}{V_0}\right), \tag{2}$$

where V_0 is the cake volume at $t = 0$. It is related to the porosity ϕ by the following equation:

$$\epsilon = \ln\left(\frac{1-\phi_0}{1-\phi}\right), \tag{3}$$

where ϕ_0 is the porosity at $t = 0$.

To determine the compressibility, which is often dependent on the porosity, the porosity as a function of the solid compressive pressure must be known. In the case of one-dimensional consolidation, the local solid compressive pressure is related to the local liquid pressure by the following momentum balance, if the gravitation terms can be neglected:

$$p_l + p_s = p, \tag{4}$$

where p_l is the local liquid pressure and p is the external load.

For the bulk compressibility, the following two situations can be distinguished:

- The local porosity is a function of the local solid compressive pressure only. In this case equilibrium is reached so quickly, creep phenomena can be neglected.
- The local porosity is a function of both solid compressive pressure and time. In this situation, if the creep rate cannot be neglected with respect to the overall rate of the consolidation process, creep of the solid phase must be taken into account. Creep is defined as the phenomenon of a solid material showing time-dependent deformation at a constant stress (Mitchell, 1976; Williams, 1973, p. 79). It is a well-known phenomenon for metals and polymers.

For the modeling of cake filtration and expression it is important to know if creep of the solid phase must be taken into account. If this is the case, the situation becomes more complicated. Since the local liquid pressure decreases during expression of a wet solid, the solid compressive pressure at a fixed position in the cake is time dependent. In the case of creeping materials, the volume strain is not only dependent on the actual pressure and time, but also on the complete history of the pressure on the cake. In Figure 1 the strain of an arbitrary material is shown as a function of time, in the case of a pressure step at $t = t_1$. The strain would show a discontinuity if it depended only on the local solid compressive pressure and time, which is unrealistic. This requires an approach where the influence of the pressure history on the creep curve is taken into account.

For the measurement of the bulk compressibility, the compression–permeability cell is often used. It is an important tool for determining the bulk compressibility and permeability. The compression–permeability cell is an apparatus in which a solid–liquid mixture can be subjected to an externally applied pressure, while liquid is led through the cake at a constant pressure. In the case of negligible creep, the liquid flow becomes constant when equilibrium is reached. However, in the case of viscoelastic materials the strain is not only dependent on the local solid compressive pressure but also on time. If creep of the solid phase cannot be neglected, it is necessary to take the time dependence of the compressibility in the modeling of cake filtration and expression into account.

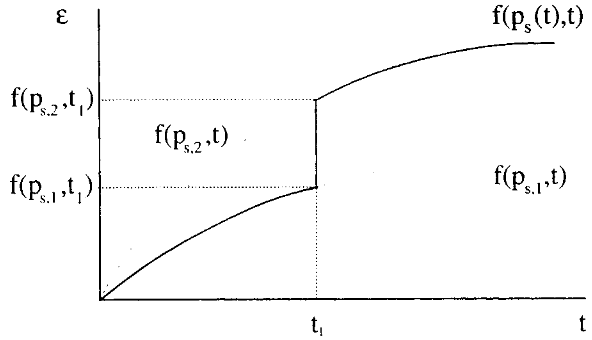

Figure 1. Strain as a function of time in the case of neglect of the pressure history.

Pressure step at $t = t_1$.

Another method to determine the bulk compressibility is by measuring the local liquid pressure and the local porosity (Chase et al., 1990; Chase and Willis, 1992). The local liquid pressure is related to the local solid compressive pressure by Eq. 4. The local liquid pressure can be measured by pressure probes, and the local porosity can be determined from cake-conductivity data. For determining the compressibility with this method no local consolidation equilibrium is necessary, thus allowing the measurement of fast creep phenomena.

In this article we report the results of the compressibility measurements of palm-oil filter cakes. First, we investigate whether conventional techniques could be applied. Since this did not appear to be the case, we eventually determined the compressibility from creep-curve measurements, which were measured using a novel technique. We also present the experimental and modeling results of the effect of the pressure history on the creep curve.

In a subsequent article the results in this article will be combined with permeability measurements, leading to a model for the expression of palm-oil filter cakes.

Models for Creep at Time-Dependent Pressures

The models for creep that are mentioned in the literature can be subdivided into linear and nonlinear viscoelastic models. A material is defined as linear viscoelastic if the strain, ϵ, is proportional to the pressure, p_s, at any time, t. Linear viscoelastic models are usually based on combinations of rheological elements. These elements are the linear spring and the linear dash pot.

Linear viscoelastic materials

If the material is linear viscoelastic, the creep curve at a time-dependent pressure can be described by the Boltzmann superposition principle (Findley et al., 1976, pp. 82–85):

$$\epsilon(p_s(t),t) = \int_0^t \frac{\partial \epsilon}{\partial p_s}(p_s, t-\xi)\cdot\frac{dp_s}{d\xi}\cdot d\xi, \tag{5}$$

where

p_s = solid compressive pressure, Pa

t = time, s

ξ = dummy variable, varying from 0 to t, s

Using the Boltzmann superposition principle, the strain can be calculated at a given time as a function of an arbitrary pressure history, based on the strain–pressure–time relationship at constant pressures ($\epsilon = f(p_s, t)$).

Nonlinear viscoelastic materials

The Boltzmann superposition principle cannot be applied to nonlinear viscoelastic materials. In this case, there is no general theory that describes the creep as a function of the pressure history. The following models for creep at time-dependent pressures are reported in literature:

Modified Superposition Method (Findley et al., 1976, pp. 229–231). Similar to the Boltzmann superposition principle, this method results in an integral form, describing the strain at a given time as a function of an arbitrary pressure history. Under a constant pressure, the nonlinear pressure–strain–time relationship can be represented by

$$\epsilon = f(p_s, t). \tag{6}$$

The modified superposition method is based on the following assumptions:

- A stepwise pressure change from $p_{s,1}$ to $p_{s,2}$ at time t_1 can be considered as the removal of pressure $p_{s,1}$ and the simultaneous application of pressure $p_{s,2}$, both being considered as independent actions.
- The recovery strain ϵ^r, resulting from a stepwise removal of $p_{s,1}$ at time t_1, can be represented by the following equation (see also Figure 2), with $t_0 = 0$:

$$\epsilon^r(p_s(t), t) = f(p_{s,1}, t - t_0) - f(p_{s,1}, t - t_1). \tag{7}$$

Using these assumptions the strain as a function of time can be described as follows, if the pressure is increased stepwise from $p_{s,1}$ to $p_{s,2}$:

$$\epsilon(p_s(t), t) = f(p_{s,1}, t - t_0) - f(p_{s,1}, t - t_1) + f(p_{s,2}, t - t_1). \tag{8}$$

For N stepwise pressure changes this equation becomes, with $p_{s,0} = 0$, $t_0 = 0$, and $\epsilon(p_{s,0}, t_0) = 0$,

$$\epsilon(p_s(t), t) = \sum_{i=0}^{N} [f(p_{s,i+1}, t - t_i) - f(p_{s,i}, t - t_i)]. \tag{9}$$

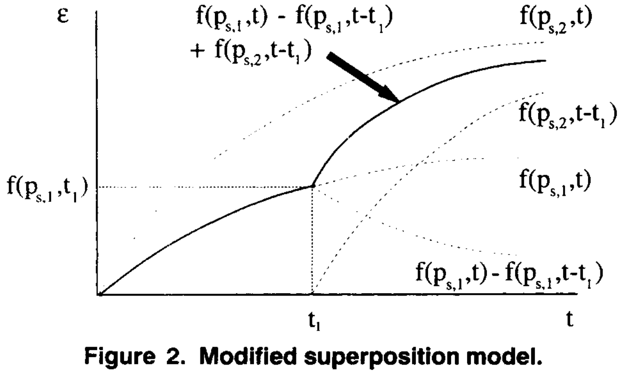

Figure 2. Modified superposition model.

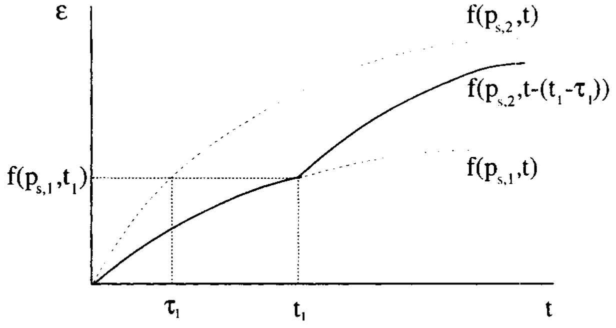

Figure 3. Strain-hardening model.

Strain-Hardening Model (Skrzypek and Hetnarski, 1993). This model assumes the following unique relationship for the creep strain rate at a constant temperature:

$$\frac{\partial \epsilon}{\partial t} = g(\epsilon, p_s). \tag{10}$$

If the pressure is increased stepwise from $p_{s,1}$ to $p_{s,2}$ at time t_1, the strain as a function of time at pressure $p_{s,2}$ can be obtained by translation across the time axis:

$$\epsilon(p_s(t), t_2) = f[p_{s,2}, t_2 - (t_1 - \tau_1)], \tag{11}$$

where τ_1 is the time needed to obtain the strain $\epsilon(p_1, t_1)$ at pressure p_2 (see also Figure 3). For N stepwise pressure changes, Eq. 11 becomes, with $\tau_0 = 0$,

$$\epsilon(p_s(t), t) = \sum_{i=1}^{N} [f(p_{s,i}, t_i - t_{i-1} + \tau_{i-1}) - f(p_{s,i}, \tau_{i-1})]. \tag{12}$$

It should be noted that $f(p_{s,i-1}, t_i) = f(p_{s,i}, \tau_i)$ (see Figure 3).

Experimental Procedure

The experiments were carried out with refined palm oil. The oil was crystallized by batch in a 4-L stirred vessel. The end temperature of the crystallization step was 28°C, leading to a mass fraction of solids of 10–11%. The crystallization conditions were kept identical for each experiment. The mass fraction of solids of the cake was measured with a Bruker minispec low-resolution pulse nuclear-magnetic-resonance (NMR) apparatus (type pc120), which is a common apparatus for the determination of the mass fraction of solids of edible fats.

To study the compressibility and permeability of the cake, compression–permeability experiments were carried out. The compressibility cell had a diameter of 80 mm and a height of 50 mm. The mass of filtrate was recorded as a function of time.

Measurement of liquid pressure and porosity gradients, from which the compressibility can be determined, is not possible for palm-oil filter cakes. The solid phase would be pressed into the pressure probe, thus leading to blockage, and the liquid phase is an electrical insulator. Another method to measure the compressibility is to prevent the development of a liquid pressure gradient by expressing dry material. If the solid compressive pressure is known and is constant across the cake, the creep curve follows from the cake thickness as a function of time. For this purpose, after filtration of the slurry, the palm-oil particles were suspended and washed with a detergent solution. The principle of this procedure is emulsifying the liquid phase, while the solid phase remains unchanged (Poot et al., 1975). After detergent washing, the particles were rinsed with water and ethanol, followed by air drying.

The complete procedure was carried out in a constant temperature room at 28°C to prevent melting or crystallization effects. The washed particles, which were not completely free of liquid, were subjected to expression experiments with the equipment shown in Figure 4. The perspex column had a diameter of 60 mm and a height of 200 mm. The displacement of the piston was recorded as a function of time. The pressure on the cake can be made an arbitrary function of time, by means of the computer-controlled throttle valve. The following methods were used to verify whether, because of the oil that was not removed by washing, there was a liquid pressure gradient during expression.

- Measuring the porosity gradient of the cake after expression. The porosity gradient was measured by cutting the cake in slices, in order to determine the solid phase content. A minimal slice thickness of 0.4 mm could be reached.
- Studying the influence of the initial cake thickness on the porosity as a function of time during expression.

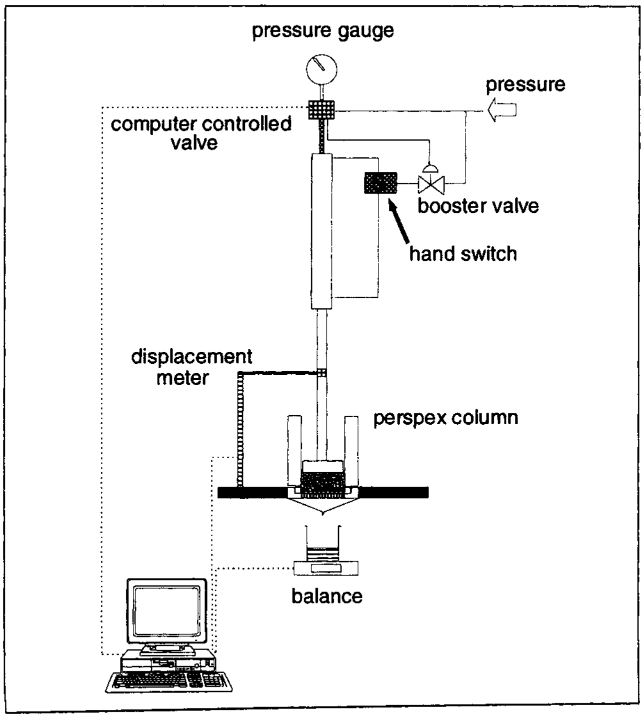

Figure 4. Equipment for creep curve measurements.

If there is no liquid pressure gradient, then there should be no gradient of solid-phase content across the cake. The initial cake thickness should have no influence on the porosity.

The porosity as a function of time was studied at various constant and time-dependent pressures. To avoid recovery effects, the pressure was not decreased during the experiments.

Results at Constant Pressures

Experimental results

In Figure 5 the flow rate as a function of time is shown for two compression-permeability experiments carried out at identical conditions. It appears that after 29 hours of expression at 1.9 bar the flow rate was not constant yet, from which it follows that no equilibrium consolidation is reached. Consequently, no equilibrium permeability and compressibility data can be obtained using this procedure.

After washing with detergent the particles were not completely free of liquid, and the solid-phase content varied from 56.7 to 70.5%. The solid-phase content before washing varied from 32 to 35%. During washing a part of the oil was replaced by air, thus maintaining the porous structure of the particles. Therefore, the liquid remaining after the washing procedure does not always lead to a liquid pressure gradient during expression. In Figure 6 the mass fraction of solids is shown as a function of the position in the cake after expression at 3.46 bar for 1.5 h. No liquid was expressed during this experiment. At the filter there is no higher mass fraction of solids than in the rest of the cake, which agrees with the absence of liquid expression. Moreover, it appears that there is a small gradient of the mass fraction of solids in the direction of the piston. This is probably caused by wall friction of the piston or the cake. The effect of wall friction is small, however, and will be even smaller in the case of liquid-saturated cakes, due to the lubricating effect. Figure 7 shows the influence of the initial cake thickness on the creep curve. From this figure it appears that there is no significant effect of the initial cake thickness on the creep curve. Further it appeared that for every experiment where liquid was expressed, there

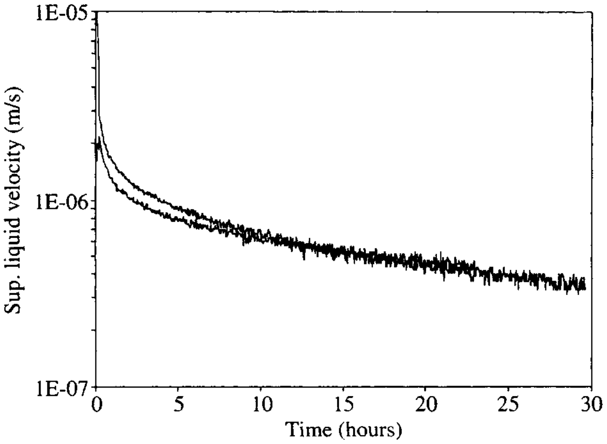

Figure 5. Liquid velocity as a function of time for compression permeability experiments.

At $p = 1.9$ bar and $p_l = 0.3$ bar (duplicate).

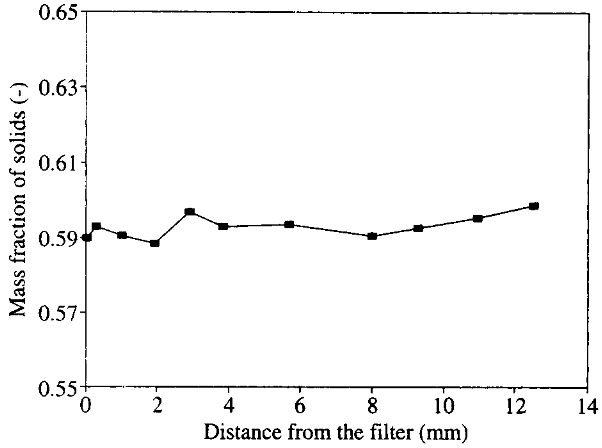

Figure 6. Porosity gradient after compression of detergent-washed particles.

At 3.46 bar for 1.5 h.

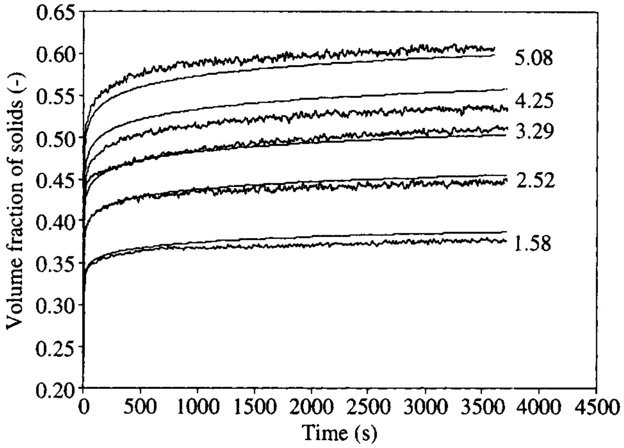

Figure 8. Experimental creep curves at various pressures (in bar) with the modeled curves using Eq. 19 (thin lines).

was no air left in the cake, and that there is no significant influence of the applied washing conditions and detergents on the creep curve. From these results, it can be concluded that there are no liquid pressure and porosity gradients if there is air left in the cake and, consequently, the measured creep curves are representative for the creep of a cake of pure solid material.

During washing part of the liquid is replaced by air. If the structure of the solid matrix does not change, the volume fraction of solids during creep-curve measurement is representative for the creep behavior in a conventional expression experiment. Therefore creep-curve measurements were performed at various pressures.

Figure 8 shows the influence of pressure on the volume fraction of solids, as a function of time. The pressure was kept constant during the experiments. At every pressure the volume fraction of solids at $t = 0$ is about 0.2. From Figure 8 it appears that the volume fraction of solids is strongly dependent on the pressure. At a pressure of 5.08 bar a solids volume fraction of 0.61 is reached within 1 hour, which is the volume fraction where all the air is expressed from the cake, and liquid expression starts. Consequently, no information about the creep curves is available at solids volume fractions higher than 0.61. Further, it appears that the volume fraction of solids has not reached its equilibrium value after 1 hour at pressures of up to 5.08 bar, and that creep is an important phenomenon during the compression of palm-oil filter cakes.

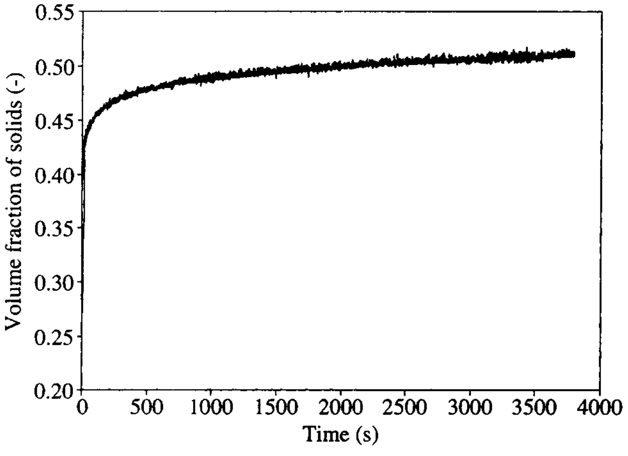

Figure 7. Influence of cake thickness (5.99×10^{-3}, 7.57×10^{-3}, and 10.8×10^{-3} $m^3_{solids}/m^2_{filter}$) on the creep curve.

Externally applied pressure: 3.5 bar.

Modeling results

For linear viscoelastic materials the volume strain ϵ is proportional to the solid compressive pressure p_s at any time t. Figure 9 shows the ratio of the strain and the pressure (ϵ/p_s) as a function of time for various pressures. If the material is linear viscoelastic, these curves should coincide. However, it appears that the ratio of the strain and the pressure depends on pressure. The strain is less dependent on pressure than for linear viscoelastic materials. Therefore, it can be concluded that palm-oil filter cakes do not show linear viscoelastic behavior, and models consisting of linear springs and

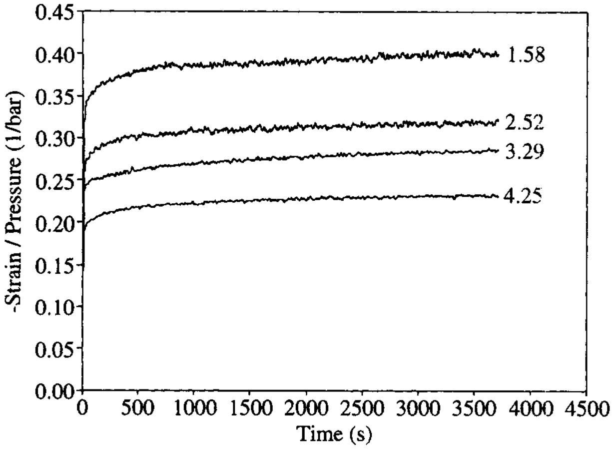

Figure 9. Quotient of strain ϵ and pressure p_s for creep curves at various pressures (in bar).

dashpots cannot be used. Another shortcoming of these models is that the strain can exceed the value that corresponds with a solid phase content of 100%.

Most nonlinear viscoelastic models mentioned in the literature are empirical. One of these empirical equations is the equation of Nutting (Nutting, 1921):

$$\epsilon = C p_s^m t^n, \tag{13}$$

where C, m, and n are empirical parameters. This equation was originally used to model the creep of asphalt and pitch. Because in this equation the strain is a power function of pressure and time, it can exceed the value that corresponds with a solid-phase content of 100%. Therefore, extrapolation to high pressures and long times will lead to incorrect results.

In the case of the compression of filter cakes there is always a maximum value of the strain, which is reached when the volume of the cake is equal to the incompressible volume. The following equation suggests a modified definition of the strain, where only the compressible volume is taken into account (see also Eq. 2):

$$d\zeta = \frac{d(V - V_s)}{V - V_s}, \tag{14}$$

where V_s is the incompressible volume (m^3), and ζ is the modified strain. At practical pressures the pure solids can be considered to be incompressible. Integrating Eq. 14 with $\zeta = 0$ at $V = V_0$ leads to

$$\zeta = \ln\left(\frac{V - V_s}{V_0 - V_s}\right), \tag{15}$$

where V_0 is the volume at $t = 0$ (m^3). The porosity ϕ is defined as

$$\phi = \frac{V - V_s}{V}. \tag{16}$$

The void ratio e is defined as follows:

$$e = \frac{\phi}{1 - \phi}. \tag{17}$$

Combining Eqs. 15 to 17 leads to:

$$\zeta = \ln\left(\frac{e}{e_0}\right), \tag{18}$$

where e_0 is the void ratio at $t = 0$. Based on the modified strain ζ, the following modification of Eq. 13 is suggested:

$$\zeta = a\left(\frac{p_s}{p_{s,\mathrm{ref}}}\right)^b \left(\frac{t}{t_{\mathrm{ref}}}\right)^c, \tag{19}$$

where

$p_{s,\mathrm{ref}}$ = reference pressure (equal to 1 bar)
t_{ref} = reference time (equal to 1 s)
a, b, c = empirical parameters

The constants $p_{s,\mathrm{ref}}$ and t_{ref} are introduced to make the equation dimensionless, and have therefore no physical background. According to Eq. 19, the porosity approaches zero if the pressure or the time approaches infinity, probably leading to better results in the case of extrapolation. Figure 8 shows the modeling results. The optimal values of the parameters are

$$a = -0.489 \pm 0.014;$$

$$b = 0.563 \pm 0.0262;$$

$$c = (4.65 \pm 0.30) \cdot 10^{-2}.$$

Results at Time-Dependent Pressures

Figure 10 shows a creep curve at a quadratically increasing pressure. This figure also shows the values predicted by Eqs. 9 and 12, using the modified equation of Nutting (Eq. 19). For comparison, modeling results are added for the case where the effect of the pressure history on the creep curve is neglected. It appears that the experimental creep curve is approximated best by the strain-hardening model. The modified superposition model leads to more or less the same results as neglect of the pressure history. Figure 11 shows modeling results with the strain-hardening model of a creep curve at a square-root-wise increasing pressure. Also from this figure it appears that the strain-hardening model gives a better description of the creep behavior of palm-oil filter cakes than neglecting the influence of the creep history. At a linearly increasing pressure, the deviations are between those at a quadratically and a square-root-wise increasing pressure. Application of the strain-hardening model instead of neglecting the pressure history heads to a reduction of the error by a factor of approximately 2.

Since creep models at time-dependent pressures require a model for creep at constant pressures, it is possible that the quality of the fit of the latter model influences the modeling results at time-dependent pressures. This effect must therefore be verified. There is a small probability that an experimental creep curve at a given pressure and time lies outside the strain range defined by the parameters $a \pm \delta a$, $b \pm \delta b$,

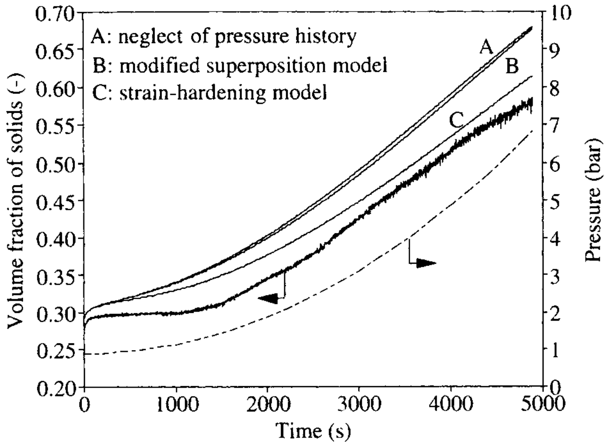

Figure 10. Experimental and modeled creep curves with different models (solid lines) at a quadratically increasing pressure (dashed line).

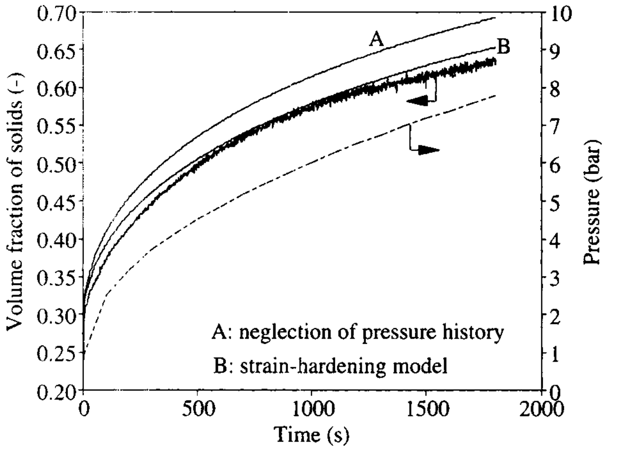

Figure 11. Experimental and modeled creep curves (solid lines) at a square-root-wise increasing pressure (dashed line).

and $c \pm \delta c$. From analysis of Eq. 19 it appeared that the calculated value of e is most dependent on the value of p_s. Therefore, at a given pressure and time the lowest void ratio is obtained at a value of $b + \delta b$, and the highest void ratio at a value of $b - \delta b$. Figure 12 shows the modeled creep curves, using the strain-hardening model, at the minimum and maximum values of b. From this figure it follows that the experimental volume fraction of solids is always outside the range defined by the parameter $b \pm \delta b$. Consequently, the strain-hardening model deviates significantly from the experiment, which cannot be explained by the quality of the fit of the creep curves at constant pressures.

From Figures 10 and 11 it appears that there is a significant effect of the pressure history on the creep curve, leading to a lower volume fraction of solids than is expected if no effect of the pressure history exists. The effect of the pressure history can be explained by the lower average pressure compared to experiments at a constant pressure, leading to a lower volume fraction of solids.

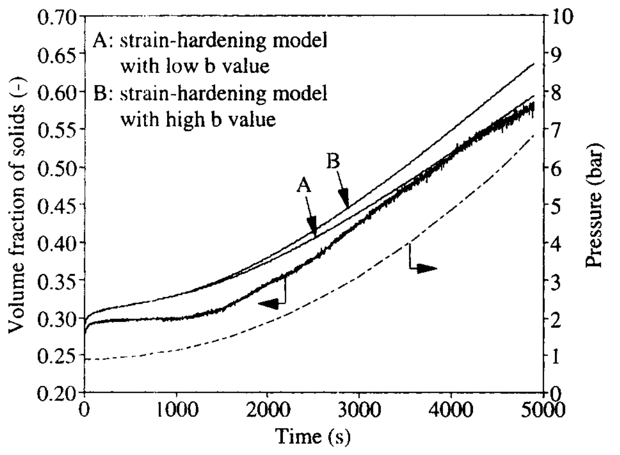

Figure 12. Experimental creep curve vs. strain-hardening model.

Using the lowest and the highest boundary of the fit values of the model for creep at constant pressures.

The deviation of the modeling results with the modified superposition model from the experiments can be explained by the assumptions of this model. From the experiments it appeared that recovery of the cake after release of the pressure is far from complete. The recovery behavior therefore, cannot be considered viscoelastic. The agreement of the modeling results of the modified superposition model with those from the neglect of the pressure history is probably caused by the slopes of the creep curves at constant pressures. The slopes are initially high, but become low for relatively short periods (about 10 s). This causes $f(p_{s,2}, t - t_1)$ to approach $f(p_{s,2}, t)$ (see Figure 2), which is the strain in the case of neglect of the pressure history.

The strain-hardening model assumes that the creep rate is a unique function of pressure and porosity. The physical background of this assumption is that the creep rate at a given pressure and porosity is independent of how this porosity has been obtained. The cake structure, which determines the creep rate at a given pressure, is considered to be a function of porosity only. From Figures 10 and 11 it appears that the strain-hardening model deviates significantly from the experiments (maximally with the volume fraction of solids of 0.04), from which it follows that the cake structure is also a function of the pressure history. There is no general theory for creep of nonlinear viscoelastic materials at time-dependent pressures. Other models mentioned in the literature are based on a multiple integral representation (Findley et al., 1976, pp. 220–238) or are physically unrealistic. The time-hardening model (Skrzypek and Hetnarski, 1993) assumes no relation between the creep rate and the porosity. The models based on a multiple integral representation are only applicable at very restricted conditions (Williams, 1973, pp. 110–112), and the accuracy is of the same order as the accuracy of single-integral models (Findley et al., 1976, pp. 237–238; Williams, 1973, pp. 110–112). Therefore, the use of more complicated models than the models described in this article does not seem to be of great practical relevance.

Conclusions

In this work, measurement and modeling results of the creep of palm-oil filter cakes are presented. The measurements were carried out by recording the cake thickness as a function of time, during expression of material from which the liquid phase was partly removed. This material was obtained by washing the filtered particles with a detergent solution. From the creep measurements, it became clear that no consolidation equilibrium was reached after 1 hour at pressures between 1.58 and 5.08 bar, which means that creep is an important phenomenon during the compression of palm-oil filter cakes. This also appeared from compression–permeability experiments after 29 hours of expression at 1.9 bar. The creep curve is strongly dependent on pressure.

The creep curves of palm-oil filter cakes cannot be modeled with linear viscoelastic models. Modeling with a modified form of the empirical Nutting equation gives satisfactory results. In the original Nutting equation the modeled porosity can become negative at high pressures and long times.

During creep of the solid phase, the solid-phase content not only depends on the actual pressure and time, but also on the pressure history. Experimental creep curves at time-dependent pressures were modeled with two nonlinear viscoelastic models, which describe the time-dependent creep behavior as a function of the pressure history and the creep curves at constant pressures. From this it followed that modeling with the strain-hardening model leads to the best results, with an absolute deviation from the experiment of 0 to 0.03, while the modified superposition model shows an absolute deviation of 0.03 to 0.07 in the volume fraction of solids.

The strain-hardening model is a good engineering tool, with which the error in the volume fraction of solids that is made by neglecting the influence of the pressure history can be reduced to 0.00–0.03, which improves the accuracy by a factor of approximately 2.

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